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Multicurve

How banks and market actors manage the liquidity crisis consequences on interest rate derivatives

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1 Document Summary

1.1 Purpose

The recent credit crisis has led institutions to adopt new counterparty risk mitigant techniques.

One of these consists in trading with a third-tier central counterparty: the trade is periodically market-to-market and collateral amount is posted accordingly by the party with the negative MtM. This way of doing has become so popular that the market now quotes collateralized interest rate swaps.

This process has to be taken into account at pricing time: The collateral is funded on a daily basis so that this OIS funding curve becomes the trade discounting curve.

The point becomes a little bit tricky when the nature of the collateral has to be taken into account. For example a trade denominated in € may be collateralized in $: Using the standard € OIS curve would not reflect the fact the collateral is posted in $ and ignore the basis spread between the two currencies. A new OIS curve ("€ collateralized in $") has to be used.

The purpose of this document is to describe how this new market standard impacts banks systems in terms of curve generation process, trade booking, hedging.

1.2 Version Control

Action	Name	Date	Notes	Version
create	Ben	06/02/2012		1.0

Valid action types are:

- Create
- Modify
- Submitted

2 The single curve methodology

Financial institutions, software houses and practitioners have developed their own proprietary methodologies in order to extract the yield curve term structure from quoted prices of a finite number of liquid market instruments : deposits, Forward Rate Agreements (FRA), short futures and standard Interest Rate Swaps (IRS).

2.1 Bootstrapping

"Best-fit" algorithms assume a smooth functional form for the term structure and calibrate its parameters in order to minimize the repricing error of the chosen set of calibration instruments. In practice, "exact-fit" algorithms are often preferred ; they fix the yield curve on a time grid of N points (pillar) in order to exactly reprice pre-selected market instruments. The implementation of such algorithms is often incremental, extending the yield curve step-by-step with the increasing maturity of the ordered instruments, in a so called "bootstrap" approach :

- Select one finite set of the most liquid vanilla interest rate instruments traded in real time on the market with increasing maturities. For EUR market, we usually consider short-term EUR Deposit, medium-term Futures on Euribor-3M and long-term Swap on Euribor-6M;
- Build one yield curve using the selected instruments plus a set of bootstrapping rules (pillars, priorities, interpolation method, date rules...);
- Compute on the same curve forward rates, cashflows, discount factors and work out the prices by summing up the discounted cashflows.

2.2 Unique curve

All derivative valuations are usually performed assuming a single standard curve. This methodology is based on the belief all market participants have equal credit risk, the firm can fund itself with LIBOR and that the embedded credit risk for rates of different maturities is negligible. This methodology makes pricing easy, since calculation libraries, systems, and reports only require one curve. Trading is simple, as the hedging process require only one set of curve perturbation.

In this single-curve approach, an unique curve is built and used to price and hedge any interest rate derivative on a given currency; this leads to the hypothesis that there exist an unique fundamental underlying short rate process able to model and explain the whole term structure of interest rates of any tenor.

3 The credit and liquidity crisis

During the credit crunch, market quotes of forward rates and zero-coupon bonds began to violate the usual non-arbitrage relationships, both under the pressure of a liquidity crisis, which reduced the credit lines needed to hedge unfunded products using zero-coupon bonds, and the possibility of a systemic break-down.

3.1 Consequences

During this credit and liquidity crisis, the assumption that each institution had equal credit risk was clearly invalidated. This observation was plainly shown in several market trends :

- Spread between LIBOR and overnight swap rates (like Eonia or Fed Fund rates) widened ;
- Basis spreads between LIBOR rates of different maturities (for example Euribor-3M and Euribor-6M) widened as well reflecting the increased liquidity risk and the corresponding preference of financial institutions for receiving payments with higher frequency (quarterly instead of semi-annually for example).

We assume that such situation is not completely new on the market ; non-zero basis swap spreads were already quoted before the crisis but their magnitude was very small and usually neglected.

3.2 Evolution of the methodologies & practices

The evolution of financial markets has triggered a general reflection about the methodology used to price and hedge interest rate derivatives. First, the large basis spreads observed on the market since the liquidity crisis imply that different yield curves are required for market coherent estimation of forward rates with different tenors (Euribor-3M, Euribor-6M...). Secondly, to adapt to the issue that LIBOR no longer reflects equal cridit risk, many financial institutions are now using a discount curve built from OIS.

As a consequence, market participants have recently started to move away from a single curve for both discounting and forecasting ; instead, they are using multiple curves, forecast are still based on LIBOR but are built specifically for different tenors. Then they construct discount curves based on overnight indexed swaps rates.

4 Multicurve model

The underlying reasons for the move from a LIBOR-based curve to an OIS-based standard curve are :

- The intense focus on collateral led the market to understand that the discounting methodology used to value derivatives must match the calculation of interest paid on collateral;
- During the crisis, banks refused to lend to each other because of counterparty risk.

4.1 From Collateral management to OIS-based discounting curves

When a derivative is in-the-money, the counterparty with positive mark-to-market collects collateral from the other counterparty. The rate used is a standard overnight rate, such as Fed Funds or EONIA which are considered as close to « risk free » as possible since the rates exist only for a single day. This process protects the positive counterparty in case of default. Since the in-the-money counterparty is paying interest on posted cash collateral, the counterparty is essentially funding the position with overnight rate.

As a consequence, it's is natural to present value-collateralized derivatives with a funding curve built from OIS. Such curve can be used to discount all trades, or only collateralized trades.

4.2 Multiple forecast curves

Before the crisis, one single curve was used for both discounting and projecting forward rates. Since market participants decided to use OIS curves for discounting, a forecast curve for projection is still required ; even if it still uses LIBOR rates, it has to be constructed consistensly with the OIS discounting curve.

Moreover, separate forecast curves must now be constructed for each LIBOR tenor used in floating rate derivative legs. As an example, Swaps indexed on 3M-LIBOR must use a different forecast curve than those indexed on 6M-LIBOR.

There are 2 possible ways to bootstrap forecast curves for each LIBOR tenor. The most direct way is to choose instruments that use the underlying of the proper tenor for the entire curve (for example, use market quotes of swap trades based on 3M-LIBOR in order to build the 3M-LIBOR curve). But instrument quotes may not be available (problem of liquidity); the alternative is then to strip a « base » LIBOR curve along with a spread built from basis swaps. For example, we can built a 1M-LIBOR curve from a already bootstrapped 3M-LIBOR curve using the 1M-3M basis swap quotes.

Forecast curve bootstrapping process includes to build the OIS curve first ; as a consequence, market actors should adapt softwares in order to take this dependency into account.

5 Consequences on systems and process evolution

To fully implement the changes needed to address new post-crisis market conditions, it is important to ensure that valuation methods, systems and reports can handle both a discount and a forecast curve to price interest rate derivatives.

5.1 Multiple curve at trade level

Adoption of OIS-based discount curves and multiple forecast curves is still a recent idea for market actors. Some of them manage this change directly at the trading desk level, when a new deal is booked into the system, additional tools compute the correct collateral schema and the corresponding discounting curve.

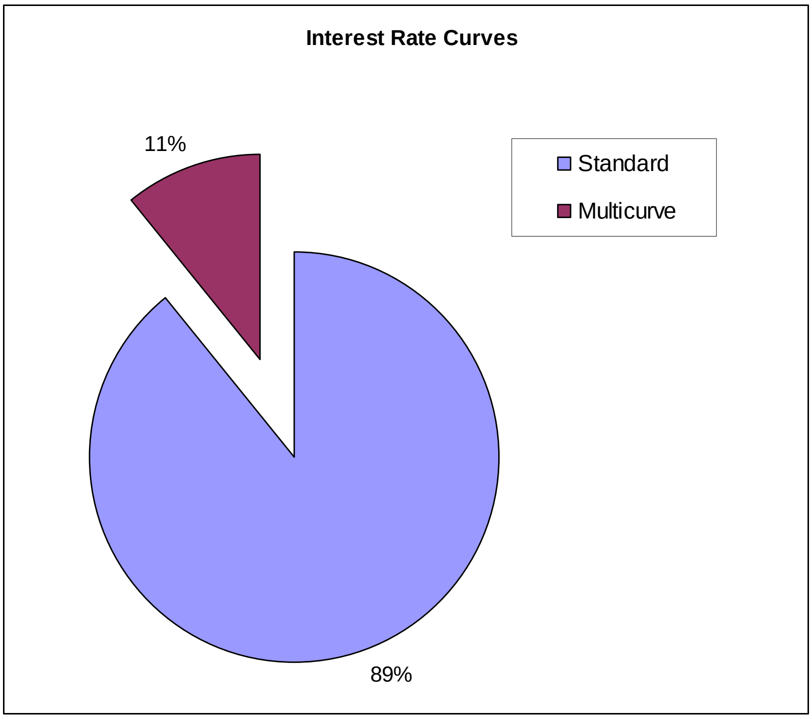
An other solution is also to keep the usual way of booking trades and apply at a given frequency (usually once per day) an automated routine which change the discount curves of all the new trades according to their collateral contracts.

Changing systems and procedures is a challenge ; softwares like Summit, Murex or Calypso have long technically supported separated discount and forecast curves at the trade level, so the evolution of such systems is not so heavy. But proprietary systems need to be adapted to accomodate multiple curves. But even if systems can technically support multiple curves, the configuration needs to be changed for each curve, as well as the pricing policies and model setup. First task is to be able to identify which trades are collateralized and which are not.

5.2 Multiple curve bootstrapping

LIBOR-based curve are now constructed using OIS curves, so the bootstrapping methodology has to be adapted in order to manage this evolution. As a consequence, any change on OIS market quotes has also an impact on the LIBOR curves depending on them ; this introduce the strong and complicated concept of curves dependency.

Any change applied on OIS curve (from any market movement or simply during a sensitivity computation) has to provoke the generation of zero-coupon rates for all the depending LIBOR rate curves ; so system should be adapted in order to keep a dependency tree. At this step, we clearly highlight that the evolution of curves bootstrapping can potentially have an important impact on the systems performance.



Moreover, market data needs to be loaded for a greater number of instrument quotes, perhaps even involving new sources.

Figure 2 : Interest rate curves in database

We see that a small number of curves are actually set in a multicurve mode

This mainly impact the 5 most important currencies : EUR, USD, GBP, JPY, CHF

5.3 Sensitivity to multicurve evolution

But more complex than system updates at the deal input level is the need to adapt middle and back office procedures to multiple curves when they were specifically designed to support a simplified single curve framework.

Middle office has to analyse market data exception and P&L explanation when multiple curves are in place ; sensitivity reports must also been updated in order to assume multiple curves.

To test the effects of the new curves on valuation, participants typically calculate sensitivities to the various curves and then estimate the effect of the new curve. After revaluing the book, the new value is compared with the estimate and any difference above a specified tolerance is investigated. The sensitivity is computed as usually, by perturbating the curve by bucket and revaluate the position in order to deduce the sensitivity at this bucket. The only change here is the fact that this computation has to be done on both curves.

5.4 The different roles of an OIS curve

As previously described, an OIS based curve used for discounting can be used at different step ; it can be used to discount trade's cashflow and also during the bootstrapping of forecast zero-coupon curve. As a consequence, the sensitivity report should also be able to distinguish the sensitivty of a deal to this curve according to its role.

One of the most important changes to apply on financial softwares is to allow sensitivity report to distinguish a trade sensitivity linked to its direct pricing or to the zero-coupon curve generation.

5.5 The impact on performances

The most challenging impact for any market participant is to be able to deploy all the mandatory changes to the softwares, libraries, reports and even the usual practices but without impacting the software performances or degrade the quality of the computation results.

Indeed, instead of considering one single curve to price a trade, we should now manage two different curves, one for forecast and one for discounting. Moreover, the forecast curve now has to be generated using an OIS based curve as discount which introduces the concept of dependency in curve construction.

As a consequence, the sensitivity reports display now more curves than in a single curve world and the computation process takes more time to produce results. Even a grid computing process has to be optimized in order to manage such increase of curves number and the new concept of curve dependency because curves cannot be generated in a random order anymore.

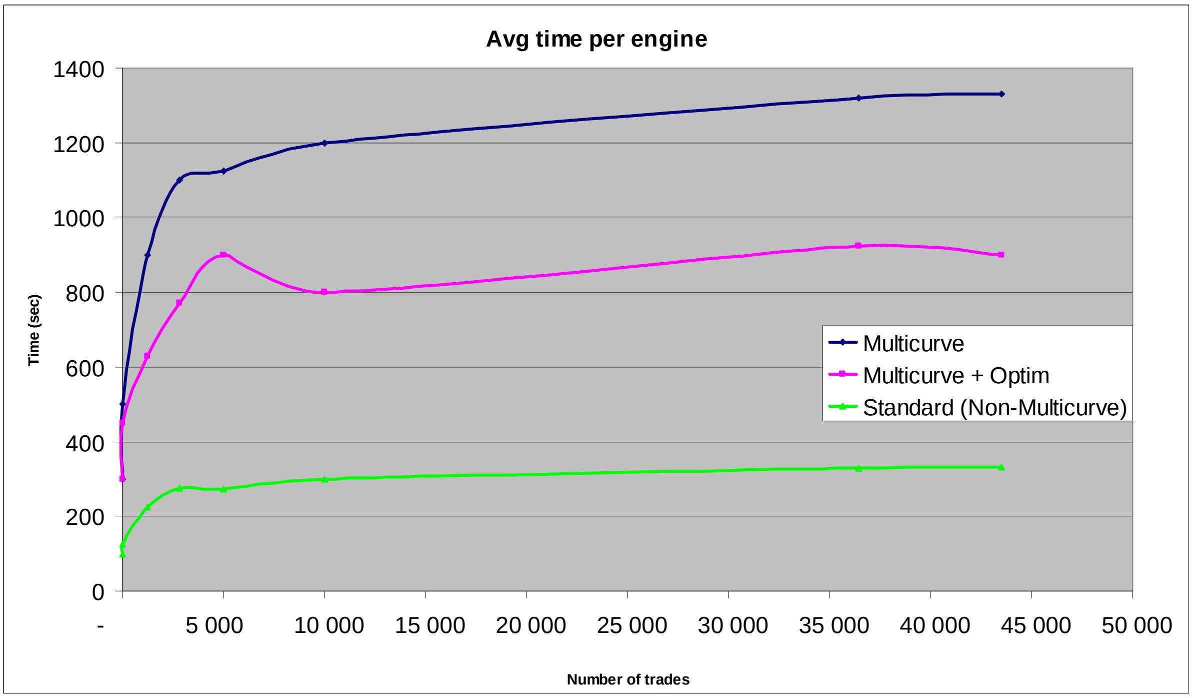


Figure 1 : Evolution of computation performances linked to the multicurve mode.

Sensitivity computation is managed through a grid computing architecture, the current graph represents the average time per engine to compute sensitivity on a pool of trades.

We see that the activation of multicurve mode has globally multiplied by 4 the average time per engine, this is due to several factors :

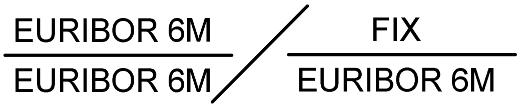
- The number of curves has increased in database
- The number of risk indexes per trade has increased because of
  - collateral management : instead of having only 1 trade for both discount and forecast, trades can have different curves ;
  - curves dependencies : shifting and regenerating an OIS based curve means that we should also generate its depending curves;

Several different types of optimisations are here available in order to reduce the impact of multicurve :

- Sensitivity optimisation : avoid shifting curve for tenors greater than trade's maturity;
- Curve generation optimisation : store the generated curves in a cache to avoid systematic and redundant bootstrapping;
- Grid computing optimisation : group trades by risk indexes in the engines in order to limit the number of different curves to generate;

6 Concrete examples

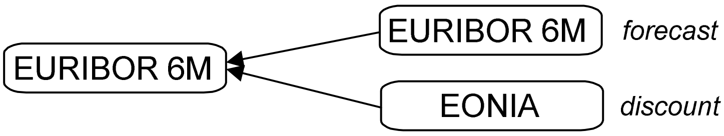
Consider a simple standard interest rate Swap trade based on EURIBOR-6M. On a non-multicurve mode, we would simply use EURIBOR6M based curve for both forecasting and discounting :



As a consequence, the current trade is sensitive to only one single curve :

Risk Type	Currency	Index
IR	EUR	EURIB6M

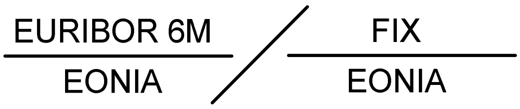
But in a multicurve mode, the EURIBOR-6M curve would need OIS based curve (in EURO market, we would consider EONIA) to build the corresponding zero-coupon rates :



As a consequence, the number of curves in the risk list is multiplied by 2 :

Risk Type	Currency	Index
IR	EUR	EURIB6M
IR	EUR	EONIA

Finally, let's assume that this trade has a collateral contract which leads to a discount on OIS, the trade definition becomes :



Of course the number of curves in the risk list doesn't change because EONIA was already present but this curve is now used at 2 different levels :

- Trade's cashlows discounting
- EURIBOR3M curve bootstrapping

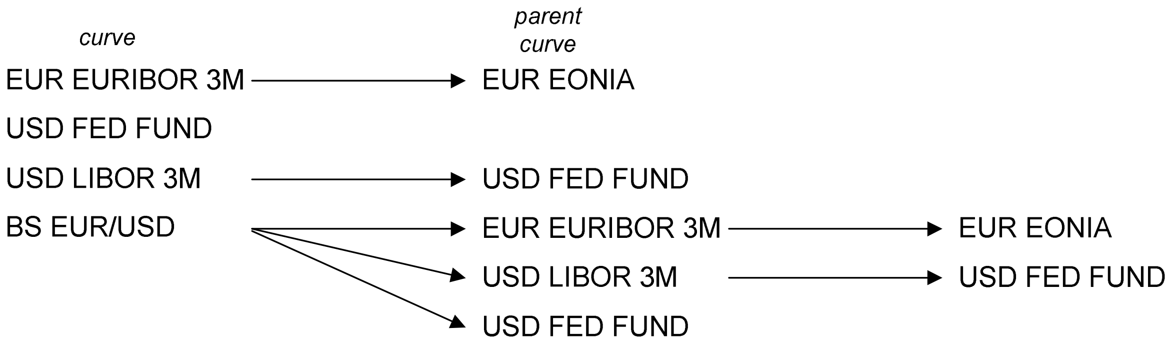
Risk Type	Currency	Index
IR	EUR	EURIB6M
IR	EUR	EONIA

Now consider a more complex case, a cross currency swap trade which collateralizes in USD. As a consequence, the EURO leg has to be discounted with a curve based on EUR/USD basis swap curve whereas the USD leg is discounted on the Fed Fund curve :

EUR EURIBOR 3M / BS EUR/USD

USD LIBOR 3M / USD FED FUND

Knowing the curves dependencies, we are able to construct the following dependency tree :



As a consequence, we easily understand that a shift applied on the EUR/EONIA curve should provoke the zero-coupon regeneration of EURIBOR3M and also the Basis curve EUR/USD. So we can have a simple representation of how complex this project can be in terms of impact on performance when the dependency tree becomes larger.

7 Conclusion

The crisis that affected financial markets in the last years leaded market practitioners to revise well known basic concepts like the ones of discount factors and forward rates. A single yield curve is not sufficient any longer to describe the market of interest rate products. But using different yield curves at the same time requires a reformulation of most of the basic assumptions made in interest rate models.

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